Measuring Distance Using Parallax

Purpose:

In this exercise you will gain familiarity with the way in which parallax measurements are used to determine distance. You will relate the accuracy of angle and baseline measurements and the length of the baseline to the accuracy with which a distance can be measured.

Historical Background:

The Greeks knew that if a distant object is viewed from two different points, its apparent direction changes. The fact that they could not measure this motion for the Sun, the stars, or the planets lead them to be uncertain of the distances to these bodies. The fact that the stars did not seem to move over the year or the day also made them doubt that the Earth moves.

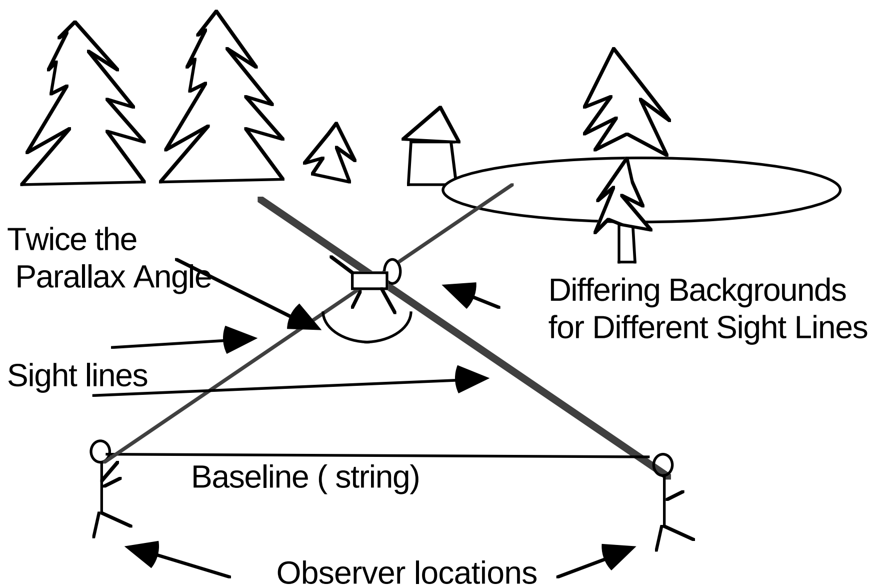
The problem is that as something gets further away, the apparent motion due to parallax gets smaller. Eventually it becomes too small to measure. (ref Astronomy Today 4th ed, sect 1.5 pp 25-27)

Materials:

Protractor, ruler or tape measure (with centimeters),calculator or spread sheet program, paper, and pencil. You may want chalk or tape to mark your location.

Procedure:

We will measure the distance to some nearby objects outdoors and then will estimate the accuracy of these measurements.



Decide on three objects to measure and baselines to use for each.

Your baseline must be **at least 200 cm long**. You **may** use the same baseline for all measurements.

Chose **3** objects based on their distances. Choose one at a distance of about **5** times the baseline, **10** times the baseline and **as far away as you dare**. You MUST be able to see the object from both ends of the baseline that you are using. (Just guess the distances as you choose the objects)/

Measure the angle between each end of the baseline and the direction to each object three times. These are the interior angles of the triangle (see the figure that Measure to a precision of **0.1 deg**. Measure the length of each baseline three times to a precision of **0.1cm** (1mm). These measurements will **not** all give the same value. The three measurements of each angle will **not** all give the same number. Be sure that you know the direction of the baseline (perhaps by drawing a line along the baseline) so that you can measure the angles properly.

Be sure to make the length measurements to the nearest millimeter (0.1 centimeter) and make angle measurements to the nearest 0.1 degrees. Repeated measurements of the same length or angle should NOT come out the same.

The point of this exercise is to experience the effects of measurement errors. So be sure to measure the baseline and each angle several times. And be sure your measurement is precise enough that the three measurements of the same baseline or angle are **not** all identical.

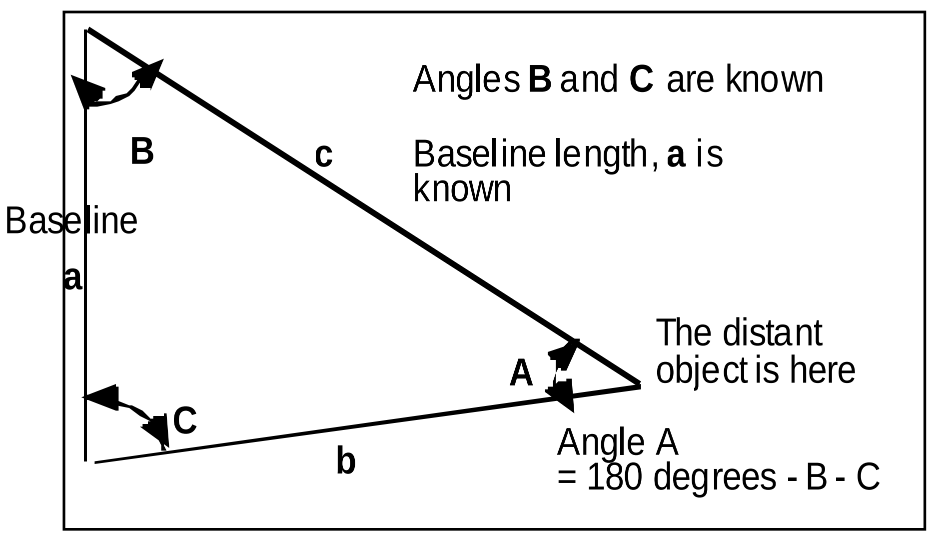
DRAW your set ups. Draw the triangle with the object at one end, the baseline opposite and lines representing the line of sight from each end of the baseline to the object. Sketch the object at the end, landmarks etc. Your picture need not be very accurate in representing the angles, but it should correctly reflect the difference between **acute** (less than 90^o) and **obtuse** (more than 90^o) angles. **Turn in the drawings**.

Measure lengths to the nearest millimeter (0.1 centimeter). Measure angles to the nearest 0.1 degrees. These will be the baseline and angles B and C as shown below,

Your measurements for each object amount to two angles and the included side (the baseline) for a triangle. This is enough information that the triangle is now determined. That is, there is only one triangle corresponding to these measurements (see the figure below). When the triangle is completed, the position of the distant object will be determined.

The “distance” to the object depends on whether the distance is defined to be from one of the observation points, from the baseline or from some other point. In astronomical measurements, Earth’s center is often used for nearby objects and the center of the Earth’s orbit is used for distant objects. For your experiment, you will find the distance from one of the observation points.

You could draw the triangle to scale and measure the distance on the scale picture. This is comparatively easy to think about, but the process of drawing introduces additional errors. We will use trigonometry instead. Don’t worry, the formulae follow.



You have measured angles B and C and the baseline (side a). Record your data in the table below or a spreadsheet laid out the same. Find angle A using:

A+B+C= 180 degrees

Be sure that the baseline is measured in centimeters to the nearest **millimeter** and that the angles are measured and computed to the nearest **1/10 degree**.

	Object # 1	Object # 2	Object #3
Name of Object			
Baseline Length Middle Value			
Largest Value Baseline Length			
Smallest Value Baseline Length			
Sight Angle B (measured) Middle Value			
Largest B Value			
Smallest B Value			
Angle C Middle Value			
Angle C Largest Value			
Angle C Smallest Value			
Angle A, Middle Value (180° minus the middle values of the other two angles)			
Angle A, Largest Value (180° minus the smallest values of the other two angles)			
Angle A, Smallest Value (180° minus the largest values of the other two angles)			

(if A comes out negative, just write it in and keep on going)

Getting the distance when you know two angles and a side

There is a theorem (called the Law of Sines) that says;

$$\frac{a}{\text{sine A}} = \frac{b}{\text{sine B}} = \frac{c}{\text{sine C}}.$$

The lower case letters represent the lengths of the sides of a triangle; the uppercase letters represent the angles. The notation " sine A" means to use a function called "sine" and to use the value for the angle A. The sine of an angle is the ratio (answer) you get when the angle is part of a right triangle and you divide the length of the side opposite the angle by the length of the hypotenuse, so the definition is

$$\sin(\text{angle}) = \frac{\text{length of side opposite}}{\text{length of hypotenuse}}$$

Once you know that a triangle is a right triangle, and you know one of the other angles, you know the shape of the triangle. This occurs because the third angle must make up the 180 degrees total for the triangle. Since all the angles are known, the shape is fixed, and so are the relative sizes of the sides. The ratio of sides, then, can also be found. Trigonometric functions just tabulate the ratios of the sides of triangles, sort of recording the shape of right triangles

To use the Law of Sines, do not use all three ratios. Use any two. For example:

$$\frac{a}{\text{sine A}} = \frac{b}{\text{sine B}}$$

Here a is the baseline, which you know. The angle B is one of the measured angles, which you know. The angle A is known because A = 180 degrees - B - C, and you known B and C both. The side of the triangle, b, is the distance we want. Rearranging

$$b = \sin B \frac{a}{\sin A}$$

$$\text{distance} = \sin B \frac{\text{baseline}}{\sin A}$$

You know A, B, and the baseline, so distance can be found.

There is a table of sine values at the back of this write up, but it is MUCH easier to use a scientific calculator or spread sheet program to find the sine of an angle. When using a calculator be sure that it is in the degrees mode. When using a spread sheet, you usually need to convert the angles from degrees to radians by dividing by 57.295… before taking the sine. Check that you are using the correct mode by taking sine(90). It comes out to 1 if you are in the correct mode.

The table following is a way to organize your work.

Example Problem:

The measurements are as follows:

Baseline = 203.4cm, Angle B = 87.3 degrees, Angle C = 84.8 degrees.

First compute angle A as

180 degrees= A + B+ C

180 degrees= A + 87.3+84.8

A = 180 degrees – 87.3 degrees – 84.8 degrees

A= 7.9 degrees

So now you are prepared to find sine A and sine B . From the table sine A = sine 7.9 degrees = 0.13744

sine B = sine 87.3 degrees = 0.99888. The sines have no units. The lengths of the two sides needed to be in the same units as one another.

Now use the formula:

$$\frac{a}{\text{sine A}} = \frac{b}{\text{sine B}}$$

$$\textbf{distance} = \textbf{sinB}\frac{\textbf{baseline}}{\textbf{sinA}}$$

and substitute for the known values.

$$\textbf{distance} = \textbf{0.99888x}\frac{\textbf{203.4cm}}{\textbf{0.13744}}$$

$$\textbf{distance} = \textbf{1478.2cm}$$

So the distance to the object is found.

As we think about the proper number of significant figures, even the value written is too precise. In this example, angle A was known to only 2 digits. While it is NOT better to round off the sines in the middle of the computation, we should be aware that we probably cannot count on the distance being known to a tenth of a centimeter.

To assess the extreme values which can result from the measurements you have made, find the distance using different combinations of the measurements. Use the table below to organize your work (or use a spreadsheet table laid out the same way), The last

could result based on the set of observations which are available, we will substitute some of the extreme values into the equations

The largest value of distance is obtained when the baseline and sine B are the largest of the possible values and when, simultaneously, sine A is the smallest. Conversely, the smallest value of the distance occurs when the baseline and sine B are the smallest and when sine A is simultaneously the largest. This combination of values is highly unlikely, but since it is the most extreme, it can be used to assess the extreme values of the distance.

Using this approach, fill in the table below. Fill in the largest and the smallest values for the baseline and for the sines. The sine of an angle between 0 and 90o gets larger as the angle gets larger. The sine of an angle between 90 and 180^o gets smaller as the angle gets larger. Be sure to set up your table so that you compute one distance using all the middle values of the sines of the angles and the distance, and the others using the values chosen to give the very largest and the very smallest value of the distance. When the table asks for largest of the sines, they will be sine of an angle nearest to 90^o, not necessarily the sine of the largest angle. (See for yourself, which angle has a larger sine, 82^o or 118^o?)

When we consider the uncertainty in a value, we might talk about the percentage error or about the variation in the error. The variation might be expressed as follows

1478.2 cm + 109cm, -170cm

The plus and minus amounts are what you might add or subtract to find the largest or smallest value likely. There are formal statistical ways to find the range of likely values, but here we will just use the range from your largest to your smallest value.

Use your measurement data to compute the distance to each body. Record your work in the following table. Be sure to write the units for each of your quantities.

Distance Formula- choose max or min values to fit formula	Dist #	Baseline Length	Angle B / Sine B	Angle A / Sine A	Distance= Baseline*($^{sineB}/_{Sine\ A}$)
Middle baseline* Middle(sine B) /Middle(sine A)	1				
Max baseline* Max(sine B) / Min(sine A)	1				
Min baseline * Min(sine B) / Max(sine A)	1				
Middle baseline* Middle(sine B) Middle(sine A)	2				
Max baseline* Max(sine B) / Min(sine A)	2				
Min baseline* Min(sine B) /Max(sine A)	2				
Middle baseline* Middle(sine B) /Middle(sine A)	3				
Max baseline* Max(sine B) / Min(sine A)	3				
Min baseline* Min(sine B) /Max(sine A)	3				

1) Which of your distance measurements has the greatest percentage of uncertainty?

(**Percentage error** = $\left(\frac{\textbf{your value - official value}}{\textbf{official value}}\right)$**x100**, use the middle value as official)

2) If you tried to measure another object which is even more distant using the same equipment which you were already using, would your answer become more or less accurate?

Distance	Best Estimate (the middle value from the previous table)	Plus (the difference between the best estimate and the largest)	Minus (the difference between the best estimate and the smallest)
#1			
#2			
#3			

3) How accurately do you think that you could measure the angles in this experiment? (a number of degrees please!)

How accurately could you determine the value for the angle A, the angle at the distant object (the one which you did not measure, but which you computed from the other two)? (a number of degrees please!)

How accurately could you measure the length of the baseline? (a number of centimeters please!)

Which of these factors, the baseline, angle A, or angle B is the largest contributor to the uncertainty in the distance which you found?

4) If you were to measure the distance to your most distant object, which set up would give you a more accurate measurement, one with a 2500 cm baseline or one with a 1000 cm baseline?

(Assume that you can get the baseline laid out with no trouble and that you are using the same equipment for both measurements.)

(Turn in Objective, Conclusion, Drawing, Three data tables, and answers to questions)

Extra Credit

The Greeks wanted to know the distance to the Moon and to the stars.

Suppose that they had stationed two persons on opposite sides of the earth, and both of the persons observed the Moon against the stars at the same time. Draw a sketch of how two people would set up this measurement so that they would be as far apart as possible.

Look up how far away the moon is. How large would angle A be?

Do you think the Greeks could have observed this change? Why or why not?

What difficulties would there have been in measuring the distance to the Moon with this method?

Sine Table

Angle, Degrees	Sine A	Angle, Degrees	Sine A	Angle, Degrees	Sine A
0	0	34	0.559192903	66	0.913545458
1	0.017452406	35	0.573576436	67	0.920504853
2	0.034899497	36	0.587785252	68	0.927183855
3	0.052335956	37	0.601815023	69	0.933580426
4	0.069756474	38	0.615661475	70	0.939692621
5	0.087155743	39	0.629320391	71	0.945518576
6	0.104528463	40	0.64278761	72	0.951056516
7	0.121869343	41	0.656059029	73	0.956304756
8	0.139173101	42	0.669130606	74	0.961261696
9	0.156434465	43	0.68199836	75	0.965925826
10	0.173648178	44	0.69465837	76	0.970295726
11	0.190808995	45	0.707106781	77	0.974370065
12	0.207911691	46	0.7193398	78	0.978147601
13	0.224951054	47	0.731353702	79	0.981627183
14	0.241921896	48	0.743144825	80	0.984807753
15	0.258819045	49	0.75470958	81	0.987688341
16	0.275637356	50	0.766044443	82	0.990268069
17	0.292371705	51	0.777145961	83	0.992546152
18	0.309016994	52	0.788010754	84	0.994521895
19	0.325568154	53	0.79863551	85	0.996194698
20	0.342020143	54	0.809016994	86	0.99756405
21	0.35836795	55	0.819152044	87	0.998629535
22	0.374606593	56	0.829037573	88	0.999390827
23	0.390731128	57	0.838670568	89	0.999847695
24	0.406736643	58	0.848048096	89.1	0.999876632
25	0.422618262	59	0.857167301	89.2	0.999902524
26	0.438371147	60	0.866025404	89.3	0.99992537
27	0.4539905	61	0.874619707	89.4	0.999945169
28	0.469471563	62	0.882947593	89.5	0.999961923
29	0.48480962	63	0.891006524	89.6	0.999975631
30	0.5	64	0.898794046	89.7	0.999986292
31	0.515038075	65	0.906307787	89.8	0.999993908
32	0.529919264	66	0.913545458	89.9	0.999998477
33	0.544639035			90	1