Chapter 7 The Start of Modern Physics

Science was not halted by the Church in 1633. The scene moved to northern Europe where the Pope had less influence. Isaac Newton (1643-1739 CE) was born in Wolstoncroft in England, to a farm-owning family. Newton's father died before Newton was born, and his mother remarried. Newton grew up on their farm, but it became clear that he would not make much of a farmer. He was sent to Cambridge University at ~18 and stayed for 4 years.

At the end of that time, in 1665, everyone was sent home from the University because the Bubonic plague was about. The only thing they knew to do to prevent the plague from spreading was to disperse people. (What does spread the plague? Could you get he plague today?)

Newton went home to the farm, and in the next 18 months figured out the physics you would study in about 3 semesters at college. He also invented the branch of mathematics, Calculus. He didn't write things down immediately, however. He was concerned that someone would steal his ideas. Only many years later, after urging by Edmund Halley, did Newton document his work in the books "Principia" and "Optics". Newton was always afraid that others would try to steal his ideas and take credit for them. When the plague abated, Newton went back to Cambridge, where his professor resigned to give Newton his job.

Newton determined the following three general laws, which definitely break with Aristotle. They are applicable to any sort of body.

<u>Newton's Laws of Motion</u> (these are the only ones called Newton's laws)

> **I. In the absence of an outside influence, a body at rest will remain at rest, and a body in motion will remain in uniform, straight line motion. (do nothing, nothing happens)**
>
> **II . Force = Mass x Acceleration**
>
> **III. For every action that a body exerts on another body, it experiences and equal and opposite force (action=reaction). (In this context, action means force. So the law says that the forces of interaction between any two bodies are equal but opposite)**

These laws are very general. They allow numerical predictions of what will happen during interactions. In a way, all three are restatements of the second law. The first law is the case of the second law when there is NO force. The third law says that when two things interact, ANY two things, they both experience the same force, but in opposite directions. It is sort of an accounting rule.

It often bothers people to think that when a bug hits your car, both the car and the bug experience force of the same magnitude. It doesn't mean that the effect on the bug and the car are the same. Which one experiences the greater effect, the bug or the car?

Conservation Laws

Newton and later physicists also figured out several more fundamental laws, called conservation laws**. A conserved quantity in physics is one that is never created or destroyed.** This is nearly the opposite of what we think of in everyday life. Because we can predict the amount of a conserved quantity, sometimes we can predict what will happen.

One might think of money as a conserved quantity. It does not appear or disappear, it is transferred from one person or account to another. So it can be worthwhile to hire an accountant to figure out what you have and where it is going. It is possible to predict how much money you have left.

On the other hand, ideas are not conserved. It if you add up the ideas of all the people in the room, it will not lead to a prediction of the number of ideas in 5 minutes.   The conservation laws are often more useful for predicting what will happen, than are Newton's laws of motion.

Conservation Laws

Linear Momentum   Mass x velocity

$$\left(m_{1\,before}\vec{v}_1 + m_2\vec{v}_2 \ldots\right)_{\text{before interaction}} = \left(m_1\vec{v}_1 + m_2\vec{v}_2 \ldots\right)_{\text{after interaction}}$$

The arrow above v for velocity indicates that its direction matters. The three dots (called an ellipsis) means that there can be more terms

Angular Momentum

Mass x Velocity in the direction around the Center x Distance to the Center

$$\left(m_1\vec{v}_1\vec{r}_1 + m_2\vec{v}_2\vec{r}_2 \ldots\right)_{\text{before interaction}} = \left(m_1\vec{v}_1\vec{r}_1 + m_2\vec{v}_2\vec{r}_2 \ldots\right)_{\text{after interaction}}$$

Angular Momentum has a magnitude and a direction associated with it. It is always conserved, but the law is most useful when there is something obviously going around something else

Total Energy

$$\left(\frac{1}{2}m_1(\vec{v}_1)^2 + PE_1 + \frac{1}{2}m_2(\vec{v}_2)^2 + PE_2 ..\right)_{\text{before interaction}} = \left(\frac{1}{2}m_1(\vec{v}_1)^2 + PE_1 + \frac{1}{2}m_2(\vec{v}_2)^2 + PE_2 ..\right)_{\text{after interaction}}$$

The total of all the energy in all the forms is conserved
Kinetic Energy   1/2 Mass x Velocity^2
Potential Energy takes many forms,
All convey the possibility of causing velocity
Gravitational Potential
Calories
Chemical Energy (like in gasoline and cooking gas)
Also, not known to Newton,

Rest Mass Energy = Mass x c^2

Why are there conservation laws? People don't decide that they should exist, the Universe does. They indicate symmetries in the structure of the Universe. There may be more conservation laws. In the laws governing how elementary particles work, there are some conservation laws governing how one (or more) particles transform into others. These laws are sometimes broken, so they have a rather different status from the laws above.

What is the use of the conservation laws? They give us some idea of what will happen in physical situations. Sometimes we know enough about a situation that the specific result of an interaction. Sometimes we can only rule out some consequences.

HOW TO USE A CONSERVATION LAW

To use a conservation law, we find all the bodies involved in an interaction or situation. Then find the amount of a conserved quantity associated with each body and add up the amount of each conserved quantity to find the total   BEFORE an interaction. AFTER the interaction, the total will be unchanged. In situations where we know some, but not all, of the quantities, the conservation law(s) allow us to find the remaining quantities.

Example:

You are driving your 1500kg car at 80 km/hr going East and you run head on into a 3000 kg truck. The truck was going west at 20 km/hr before your collision. After the collision, your car and the truck stick together. No one puts on the brakes. How fast are you (both) going after the collision?
Answer:
The information given tells the masses of the cars and their velocities. So we know enough to compute the total momentum before the collision. Adding it up

$$\text{Total Momentum}_{\text{before}} = (m_1 v_1 + m_2 v_2)_{\text{before}}$$

$$= (1500\text{kg})(80\text{km/hr}) + (3000\text{kg})(-20\text{km/hr})$$

$$= 120{,}000 \text{ kg km/hr} - 60{,}000 \text{ kg km/hr}$$

$$= 60{,}000 \text{ kg km/hr}$$

The truck's velocity and your velocity have opposite signs because they go opposite directions. The total momentum has a positive sign, because your car had more than the other car. You made the decision to call eastward positive. It really doesn't matter which direction is positive, but you need to keep track.

Now we need to figure out the velocity of each vehicle AFTER the collision. We know that if the two vehicles stuck together, they must be moving with the **same** velocity. After the collision

$$\text{Total Momentum} = 60{,}000\text{kgkm/hr}$$

$$(1500\text{kg})(v) + (3000\text{kg})(v) = 60{,}000\text{kgkm/hr}$$

$$(4500\text{kg})(v) = 60{,}000\text{kgkm/hr}$$

Dividing both sides by 4500kg yields

$$\frac{60{,}000\text{kgkm/hr}}{4500\text{kg}} = v$$

$$13.3333\text{km/hr} = v$$

Both vehicles are going the same direction at 13.333km/hr. They are both going to the EAST. You can tell because you set up the problem with Eastward velocity as positive. Since your answer is positive, it must be East.

What if you had been going the same direction (both eastward) and you rear-ended the truck? Assume that the two vehicles had the same masses and velocities as before, except you are both going East. If they stick together after the collision, how fast would you be going after the collision? (40 km/hr eastward)

Example 2
You are parked behind your friend's 2000 kg car. Your car has mass 2500kg. You mess up. You put your foot on the accelerator too hard. Then you realize what you have done. You take your foot off the accelerator, but your car runs into the other car at 4 mph. Your friend's car is sitting, parked, in neutral. If your car is going 1.5 mph after the collision, how fast is your friend's car going? Again, we can use conservation of linear momentum. Before the collision

$$\text{Total Momentum}_{\text{before}} = (m_1 v_1 + m_2 v_2)_{\text{before}}$$

$$= (2500\text{kg})(4\text{mi/hr}) + (2000\text{kg})(0\text{ mi/hr}) = 10000\text{kgmi/hr}$$

After the collision, the total momentum is the same, and the velocity of your friend's car is unknown, so

$$(2500kg)(1.5mi/hr) + (2000kg)(v) = 10000kgmi/hr$$

$$3750kgmi/hr \quad + (2000kg)(v) = 10000kgmi/hr$$

$$(2000kg)(v) = 10000kgmi/hr - 3750kgmi/hr$$

$$= 6250kgmi/hr$$

$$v = 3.125 \text{ mi/hr}$$

We know that your friend's car is going in the same direction that your car was going because the answer has the same positive sign as your original motion had.

The law of **conservation of linear momentum** is generally useful. It is equivalent to Newton's second law, but is easier to use, because we scarcely ever know the exact force between two bodies at every instant. Conservation of linear momentum adds up all the effects of the forces and allows us to know the situation after the interaction.

The law of **conservation of angular momentum** is always true (and it is possible to define the radius to any center you like), but it is most useful when something is obviously going around something else.

Conservation of energy is always true as well, but there are many formulae for the potential energy. It is often more useful as a general concept. For example, in the car-truck collision, the kinetic energy of motion of the vehicles is transformed into potential energy by crunching the metal. The amount of energy used in bumper crunching depends on how the cars are built.

Gravity

Newton also discovered the force of gravity. The story is that he was thinking about why the Moon orbits the Earth and what keeps it from getting away. Then an apple fell. He realized that a force between every object and every other one must exist, and that the form of the force law is

$$\mathbf{F} = \frac{-\mathbf{G}\mathbf{m_1}\mathbf{m_2}}{\mathbf{r^2}}$$

F = the force

G = the Universal Constant of Gravity

m_1, m_2 = the two masses involved

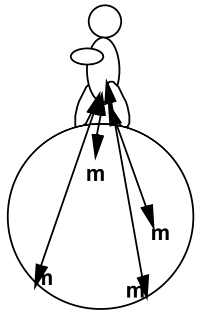
r = the distance between the masses

The negative sign in the force law indicates that the force is in the direction to pull the bodies together, the force acts along the line joining the bodies.

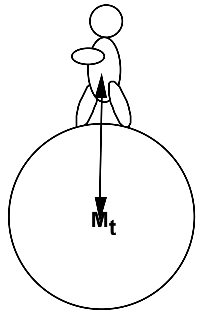
According to Newton, there is a force of gravity between every pair of bodies. Or better, between every two particles with mass. There is no minimum size of the and there is no maximum distance for gravity to act, so far as we know. So you are pulling on and being pulled on by galaxies you have never seen; ones you didn't know existed. So why don't we feel the gravitational force from each other?

The value of G, the Universal Constant of Gravity, is a small number. So the force between you and me is very small. On the other hand, gravity always attracts. So it builds up to a large value between big bodies like planets and satellites.

As we consider the force of gravity from some large body, it becomes obvious that the material is not all at the same distance. To find the total force of gravity between two bodies, one would add up the forces between every tiny piece of each. This gets old really fast.



All parts of Earth pull on the observer



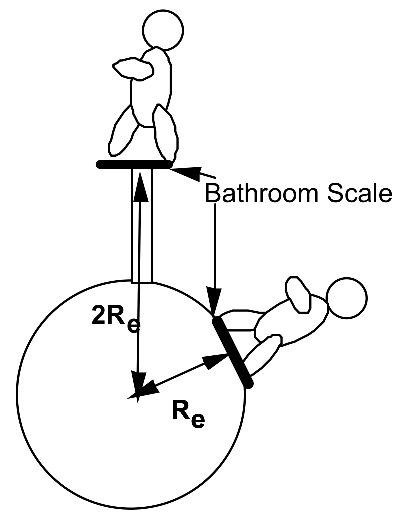
The distances and amounts of mass average to act like all mass at the center

So how did Newton compute the force between the Earth and Moon? Newton, used the Calculus (mathematics,not the tooth scum) to add up the force from the different particles in a large body. One of the things he proved was that

The force from a body with spherical symmetry produces the same gravitational force as would the same total mass concentrated all at the center.

So when we compute the force of gravity on an object on the surface of the Earth, the distance between the body and the Earth is the radius of the Earth.

Example: You weigh 150lb on the surface of the Earth.
How much would you weigh on a tower 1 Earth radius high?



The 150lb is NOT your mass, it is your weight, the force of gravity between you and the Earth. Your bathroom scale is squashed because of this force. The very idea that the weight changes indicates that 150lb is not your mass.

It is possible to use the force to figure out your mass, but it is not necessary for solving this problem. In stead, we can solve the problem by comparing the situation at the lower and the higher positions. This method simplifies the computation,

Start by writing out the force of gravity on you standing on the Earth.

The general law is $F_g = -\frac{Gm_1m_2}{r^2}$.

Substituting the variables for this specific case (just the names, not the numbers).

$$150lb = -\frac{Gm_{earth}m_{you}}{R_{earth}^{\;2}}$$

Now set up the equation when at the top of the building, at a distance of $2R_{earth}$ from the center of the Earth.

$$F_g = -\frac{Gm_{earth}m_{you}}{(2R_{earth})^2} = -\frac{Gm_{earth}m_{you}}{(2)^2(R_{earth})^2} = -\frac{Gm_{earth}m_{you}}{4R_{earth}^{\;2}}$$

You still don't know your mass, but the right hand side has all the same terms as the force of gravity on you on the Earth. So, factoring out the parts that equal your weight produces

$$F_g = -\frac{Gm_{earth}m_{you}}{4R_{earth}^{\;2}} = -\frac{1}{4} \times \left(-\frac{Gm_{earth}m_{you}}{R_{earth}^{\;2}}\right)$$

$$= \frac{1}{4} \times 150lb = 37.5lb$$

So doubling the distance from the center of the Earth, divides the force of gravity by 2^2, or 4, and it was not necessary to know your mass, or the Earth's mass or the constant of gravity.

Test Yourself: Your dog really wants to be picked up, but at 110lb he is too heavy. You get the idea to take the dog to the top of a tower 1.5 Earth radii high, then lift him. Will this help? How much will he weigh at the top of the tower? Ans 17.6lb

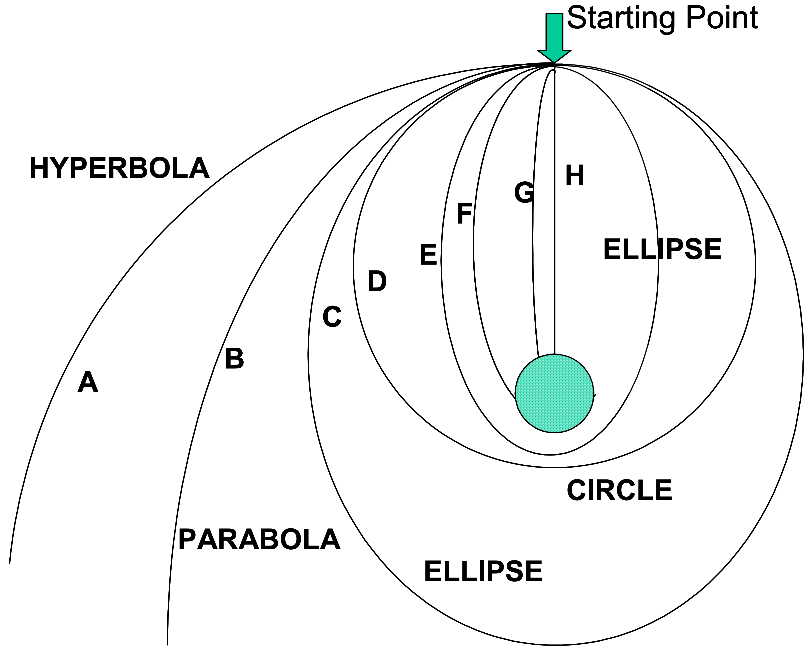
What if you want to go to Mars with your dog? What will the dog weigh? In this case BOTH the mass of the planet and the radius of the planet change. The mass of Mars is about 0.107 x mass of the Earth and the radius is about 0.53 of the Earth's mass. The dog's mass doesn't change.

$$F_g = -\frac{Gm_{dog}m_{mars}}{R_{mars}^{\ 2}} = -\frac{Gm_{dog}0.107m_{earth}}{(0.53R_{earth})^2}$$

You finish the computation. The dog will weigh 0.381 x as much as on Earth's surface = 41.9 lb

Circular and Escape Velocity

Newton discovered gravity because he was thinking about why the Moon orbits the Earth. As he worked out the problem, he discovered that an orbit would be a circle, an ellipse, a parabola or a hyperbola. These shapes are shown in the figure.



Newton found that for an object, m_1 to go in a circle around another object, m_2, a body must have exactly circular velocity compared to inertial space. The circular velocity formula is

$$V_{circ} = \sqrt{\frac{Gm_{other}}{r}}$$

For one body to escape from another, it must have escape velocity.

$$\text{Escape Velocity } V_{esc} = \sqrt{\frac{2Gm_{other}}{r}} = \sqrt{2}\ V_{circ}$$

When one body escapes from another, it has just enough velocity so that gravity slows it down forever and it finally gets to zero velocity at an infinite distance. But it is never pulled back. Again this is the speed of mass 1 compared to inertial space, not to mass 2

Newton didn't know the value of G. On the other hand, the velocity of the Moon is easily found from the distance to the Moon and the time for the Moon to complete its orbit.

To orbit the Earth at about 100 miles altitude (but r~ 6400 km, since it is measured from the center of the Earth) requires a speed of about 7.8 km/sec. The Moon, on the other hand is nearly 400, 000 km away and its speed is much less, approximately 1 km/sec.

Does this make sense? The gravity force pulls one body toward another. The crosswise motion of the objects (compared to one another) keeps them from just falling into one another. Since gravity is smaller at greater distances, the crosswise velocity needed to prevent impact is smaller.

The speed of the Earth as it orbits the Sun is about 30 km/sec. It is nearly constant, since the distance to the Sun is nearly constant. For the Earth to escape from the Sun, we would need to give the Earth only √2 times as large a velocity, or 42.3km/sec. That is just an extra 12.3 km/sec.

If an object has speed greater than escape velocity, it moves on a path called a hyperbola. It will be able to escape from the other object. It will slow down until its speed is the difference between its original speed and the escape velocity.

Example
How fast does an asteroid at 4 AU orbit the Sun (assuming that it is orbiting in a circle)?

The formula we use is the formula for the circular velocity, $V_{circ} = \sqrt{\frac{Gm_2}{r}}$ In this case, the Sun is m_2 and the distance between the centers of the Sun and the asteroid is 4AU.
It is possible to look up G, M_{sun} and then to convert 4 AU into meters. So in this case, it is possible to plug all the values into the equation for circular velocity, convert units and find out the speed. This is fun ONE time, but there is a faster way.

Since we already know the velocity of the Earth as it orbits the Sun, and the distance to the Sun, and the fact that the Earth is much less massive than the Sun, the following can be done.

First write out the circular velocity law and plug in the definitions for the Earth in orbit around the Sun.

$$V_{\text{earth orbiting Sun}} = \sqrt{\frac{Gm_{sun}}{1AU}}$$

and substituting the answer is

$$30\text{km/sec} = \sqrt{\frac{G(m_{sun})}{1AU}}$$

Write the equation for the velocity of the asteroid orbiting the Sun at 4AU

$$V_{\text{circular, asteroid orbiting Sun}} = \sqrt{\frac{Gm_{sun}}{4AU}}$$

Factoring the equation for the asteroid's velocity to isolate the terms that appear in the formula for the velocity of the Earth produces

$$V_{\text{asteroid orbiting Sun}} = \sqrt{\frac{G(m_{sun})}{4AU}}$$
$$= \sqrt{\frac{1}{4}}\sqrt{\frac{G(m_{sun})}{1AU}}$$

The term in the square root sign is something you have seen before. It is just the formula for the circular velocity of the Earth. But we know the answer, that is, the value of the Earth's orbital veloicity. So substituting for the Earth's orbital velocity yields

$$V_{\text{asteroid orbiting Sun}} = \sqrt{\frac{1}{4}}\sqrt{\frac{G(m_{sun})}{1AU}} = \sqrt{\frac{1}{4}}\,V_{\text{earth orbiting Sun}}$$
$$= \sqrt{\frac{1}{4}}\ 30\text{ km/sec}$$
$$= \frac{1}{2}\ 30\text{ km/sec} = 15\text{ km/sec}$$

What was the point? Since one result was known, we didn't have to do any complicated computations. Please notice, the asteroid is 4 times as far from the Sun as is the Earth, but the speed in orbit is half as large. The speed does fulfill our precept of " faster when closer", but the speeds are not proportional to the distance. The speed varies less strongly.

Test yourself. How fast does Pluto go in its orbit? (The answer is in the book, but be certain that you can compute the answer).

How fast would we need to make the asteroid go to get it to escape from the Sun? The asteroid would need to go $\sqrt{2}$ x$V_{circular,\ or}$ $\sqrt{2}$ x 15km/sec = 21.2132km/sec or faster
How fast do we need to escape from the Earth? Remember that you already know how fast a body would need to go to orbit the Earth near its surface.

The formula for escape velocity is found based on the idea one body escaping from another with nothing else around. In real life, there are always other objects and the effect of their gravity must be considered as well.

Can we always escape? We, you and I, don't personally have the velocity. That is what we build rockets for. But even with the best conceivable rocket, escape is not always possible.

We think that the fastest anything can go is c, the speed of light, 300,000km/sec. So what if the escape velocity $V_{esc} = \sqrt{\frac{2Gm}{r}}$ equals or exceeds the speed of light? Then we think that nothing can escape and we term the situation **a "black hole".** The place where the escape velocity equals the velocity of light. This is called the **event horizon**.

When we are far away from a black hole, we experience normal gravity. We are not sucked in. If you get near the region where the escape velocity reaches the speed of light, eventually an orbiting body will spiral in. If something crosses the event horizon, it cannot escape and no signal from it can come out.

You might imagine the black hole like the toilet. It isn't at all dangerous from far away, but once you get too far into the bowl, too bad.

Black holes can come in any size. We have already discovered black holes which appear to be the remains of a massive star. The part that is left has perhaps 7 times the mass of the Sun. These black holes are detected by the fact that they are in a system with another star and the other star orbits the black hole (far enough away that it is not sucked in).

Another type of black hole we have discovered lives at the centers of galaxies. In this case the black holes have millions to billions of times the mass of our Sun. We are not certain how they got the mass, but probably they have been swallowing stars for billions of years. The center of our own Milky Way includes such a black hole. Larger galaxies have more massive black holes.

How can we find a black hole anyway if nothing comes out of it?
Basically we need to observe something nearby. If there is something orbiting the hole under the influence of its gravity (but not already sucked in), we can use the circular velocity law to find the mass in the hole. Remembering that $V_{circ} = \sqrt{\frac{Gm_2}{r}}$, we would observe the velocity, V_{circ}, and the distance between the bodies, r. It is not really necessary to see both bodies, since the size of the orbit can be seen from the motion of just one, The value of G is known. So everything is known except m_2. So we solve for the sum of the mass of the other object. If the mass is large and there is no luminous body visible, it is possible to infer that there is a black hole.

This method, measuring V_{circ} and r, and using the circular velocity law to find the mass, is the way we measure the mass of all celestial bodies. If the laws of physics or G are different elsewhere or at other times, we have the wrong values for masses of most celestial objects.

Practice Problems

1) How far would you have to go to feel no force from the Earth's gravity?

2) You hit two different golf balls off the tee. You hit both exactly the same, but one has twice the mass of the other. Which leaves the tee faster? What law (s) of physics are useful here?
3) You are throwing a dodge ball against a grocery cart. The dodge ball bounces back to you. What will the cart do? What law (s) of physics are helpful for finding the answer?
4) When you throw a ball in the air, it comes to a halt and returns. What is the kinetic energy at the highest point of the ball's trajectory? Is this the same kinetic energy as when the ball leaves your hand? If not, what happened to the energy?
5) You buy a 5 lb sack of rice at sea level, what does it weigh on top of a tower 3 earth radii high?
Answers 1)There is no place you can go 2) The one with twice the mass leaves with half the velocity. Use Newton's second law 3) Grocery cart moves away from you, conservation of momentum or Newton's 1st and 2nd laws 4) 0 at the top of the trajectory, it became potential energy 5) 0.3125lb

Using Equations and Formulae

Many students ask, “How do you know what equation to use and how do you know what to do with it?” While we don’t know what to do in every situation, it is often easy to decide which formulae to use.

As we look at the laws and formulae in this course so far, they can be organized as follows:
Kepler’s 3 laws
Newton’s 3 laws of motion
Three conservation laws
Three formulae concerning motion with gravity
In the future, we will learn another formula concerning the Doppler shift of light

Each of these laws concerns a specific subject and involves just a few variables. The first step to knowing what to use is to understand the topic (s) of the formulae and to know what the variables stand for. Once you know the topic, you can be the judge of when to use the information. For example, if your car won’t start, you look for a car manual, not for instructions on how to substitute cooking oil for butter in cookies or how to reboot your home computer. Then you look in the instructions or the topics and look for the ones that say “car doesn’t start”, not ones to help change a flat tire. Then you start looking at the symptoms, like “key doesn’t turn” or “key turns but doesn’t cause the motor to rev” etc. When you find whatever looks the closest to what you have, you try what the book suggests.

Hopefully, this sounds like basic common sense. It is, just use yours.

So the steps are to first

1) Decide the topic of the law or equation
2) Know what all the variables are and what they mean

These steps should be done as you learn the equation. They don’t depend on the problem at hand. The table on the next few pages has spaces for you to fill in the laws, their topics and variables. Leave the Doppler shift equation until after you have studied it. But do fill in the others.

So first of all, let’s break out the equations and identify what the topics are. Remember, the laws and definitions are in the chapters.

So, check yourself.

a) What topic is Kepler’s third law about?
b) When would you use Newton’s third law?
c) What is the G in some of the equations?
d) Where is r, the distance between bodies measured?
e) What can you do with Kepler’s first law? What can be computed?

So now how would you use the equations or laws? All of them are true at the same time, but they don’t always relate to the issue at hand. So look for the one or ones that matter.
Which law or laws are relevant for the following situations?

f) How much does a 2 lb package of meat weigh when it is on top of a mountain 0.55 Earth radii tall (above sea level)?
g) If you notice a comet orbiting at an average distance of 35 AU from the Sun, how long will the comet take to complete an orbit?
h) How fast must the comet from the question above go?
i) How fast would the comet from the question above need to go to escape from the Sun?
j) A comet goes from a distance of 10 AU from the Sun at the closest to a distance of 50 AU at the furthest. Where does it move the fastest?
k) A comet goes from a distance of 10 AU from the Sun at the closest to a distance of 50 AU at the furthest. Where does it experience the most force from the Sun?
l) You are shooting arrows with a bow. You always draw the bow the same amount and the bow doesn't age etc. If you have some arrows that are more massive and others that are less massive, how can you tell which is which?
m) With the bow and arrow above, the arrow should have energy after it leaves the bow. What kind of energy did it get? Where did the energy come from ?
n) If the bow from the previous question puts a force of 30 lb on the arrow, how much force does it put on the person holding the bow?
o) You rear end another car while you are going straight East. Your car has a mass of 1500 kg and the other has mass 1200kg. You were going 10 m/s before the accident and the other car was going 8 m/s to the East before the collision. Can the other car go to the North as a consequence of the collision?
p) If you are going 9 m/s after the collision from the question above, how fast is the other car going?
q) Is there any body damage in the car accident above?
r) How much will you weigh on a planet with half the mass of the Earth and half the radius? Hint: compare your new weight to your weight on Earth.
s) You hit a golf ball and give it some spin. You are at a golf practice range which is very noisy and small. So you cannot hear when you hit a ball. You are a good golfer and every ball hits a net at the end of a short practice area. If you hit every golf ball the same, is there any way to tell whether the ball is a hollow practice ball or a ball with a rubber interior?
t) Consider the golf ball from the problem above. The cover of the ball opens and the wrapped rubber band inside starts to unravel, will it spin faster or slower than before it started to unravel?

Once you have decided which law is relevant, then it MAY be possible to find a numerical answer. There are several issues to remember, however.

First, some of the laws don't give a numerical answer. Which ones? Kepler's first law doesn't. Kepler's second law can, but the computation is beyond what we are going to learn. The law of conservation of energy also doesn't give easy numerical values.

Second, some of the laws, like the law of gravity, the equations for circular and escape velocity and Kepler's third law ($P^2 = a^3$) have equations with answers that deal with one situation. Others, like conservation laws need values BEFORE and AFTER an interaction. So look for that before and after situation to know if you might used conservation

Example: You live on the planet Zonds. You wake up this morning and weigh 200 lb. The planet is having a really bad day. Over the course of the day, Zond swells(due to an excess of underground gas). Before any of the gas escapes, the surface has swollen until it is 15% larger than before. You step onto your bathroom scale before leaving for safer locations. What do you weigh?

Work: The equation to use is clearly the formula for the force of gravity $F = \frac{-Gm_1m_2}{r^2}$, since weight is a measure of the force between the two bodies. But you don't know any of the numbers except, possibly G. To plug in and evaluate the force, would be impossible. On the other hand, we do know the force of gravity, F, to be 200lb to start and we know all the definitions. So it is easy to make a ratio. The masses are the mass of you and the mass of Zonds. Neither is in any textbook. The distance, r, is the distance between you and the center of Zonds. It is r_{zonds} at the start of the day and 1.15 r_{zonds}, 15% larger at the end. So we know:

$$F = \frac{-Gm_1m_2}{r^2}$$

start of the day $200lb = \frac{-Gm_{you}m_{Zonds}}{r_{zonds}^2}$

end of the day $x = \frac{-Gm_{you}m_{Zonds}}{(1.15r_{zonds})^2}$

where x will be your weight at the end of the day after the planet has swelled up.

The problem tells you that both you and the planet have the same mass as at the start of the day. Expanding the square in the denominator produces

$$x = \frac{-Gm_{you}m_{Zonds}}{1.3225r_{zonds}^2}$$

$$x = \frac{200lb}{1.3225} = 151.23lb$$

To get to the last step, it we substituted 200lb for all the terms in the formula that produced the 200lb. There was no way to do all the substitutions one by one, because we didn't know the individual values.

In the problem above, it was clear that it would be necessary to use the force of gravity equation. What about using a conservation law?

For example: You are playing croquet with an odd set of balls. You hit your ball and give it a 1m/s going north. It hits a stationary ball which has twice the mass of yours and stops immediately. How fast does the other ball going after the collision?

Answer: Look at the equations and the information you have. You can decide by process of elimination. The information has no data concerning force. So Newton's laws of motion aren't useful. The motion of the croquet balls isn't determined by gravity. Gravity only keeps the balls on the ground. Kepler's laws are related to motion of planets. So that lets out the law of gravity and the formulae for circular and escape velocity. The conservation laws all apply, but the information is only enough for the conservation of linear momentum. So to use a conservation law:

Find the conserved quantity, the total linear momemtum, before the interaction.

Mass of your ball x velocity of your ball +Mass of other ball x velocity of other ball =
(M)(1m/s) + (2M)(0)=1Mm/s

Total linear momentum after the interaction

Mass of your ball x velocity of your ball +Mass of other ball x velocity of other ball =
(M)(0m/s) + (2M)(x)= the total momentum from before the balls hit, that is =1Mm/s
2Mx=1Mm/s
x=1/2 m/s There is enough information to solve for the final situation, after computing the total momentum and ignoring the fact that we don't know the mass of our croquet ball.

Give the problems a try. First do a – e, then look at the rest and see what law to use.
Answers follow the table.

	Law or Equation	Statement of Law or Formula	Topics	Definitions of the Variables
1	Kepler's First Law			
2	Kepler's Second Law			
3	Kepler's Third Law			
4	Newton's First Law of Motion			
5	Newton's Second Law of Motion			
6	Newton's Third Law of Motion			
7	Conservation of Linear Momentum			

8	Conservation of Angular Momentum			
9	Conservation of Energy			
7	Law of Gravity			
8	Circular Velocity Equation			
9	Escape Velocity			
	The following is coming			
	Doppler Shift Equation			

Ans Here you are deciding what law to use. a) The relationship between the period, P, the time to orbit the Sun, and the distance from the Sun as represented by the semimajor axis of the orbit, a b) When you have an interaction between two bodies. It doesn't tell how large the force is, but does say both bodies feel equal force c) The Universal Gravitational Constant, 6.67×10^{-11} $m^3/(s^2kg^{-1})$. But you won't need to know the value. d) r is the distance between the bodies producing force in the formula. It is measured from the position of the actual mass involved. In the case of spherically symmetric bodies, the effective distance of each body is at the center. e) You can decide whether a path is an ellipse and is or isn't a proper path for a planet, or other body, orbiting due to gravity. You can't compute anything with it easily. f) force of gravity g) Keplers third law h)Circular Velocity Equation i) Escape Velocity Equation j)Kepler's second law k) force of gravity l) Newton's second law m) Conservation of Energy n)Newton's Third Law o) Conservation of Linear Momentum p) Conservation of Linear Momentum q) Conservation of Energy, there is less kinetic energy after the collision, so some energy must have gone to potential, in the form or crunching the metal of the cars. r) force of gravity s) Newton's second law t) Conservation of Angular momentum

Now try to get numerical answers
Ans the numerical values f) 0.832lb g) 207.1 years h) remember that the Earth orbits at 1 AU and has a speed of 30 km/sec. Then use the circular velocity equation and take the ratio of the velocities for the two distances 1 AU and 35 AU. to get 5.07 km/s. i) (√2)circular velocity =(√2)5.07km/s=7.17 km/s j) When it is closest to the sun, at 10 AU. You haven't learned any formula for this specific speed and I am not expecting you to. But the essence of the law is conservation of angular momentum, so there are ways to figure out the speed if you feel so inclined. k) When it is closest, because of the law of gravity. Since we don't know the mass of the comet, we cannot compute the force. l) The more massive arrow accelerates less due to the force from the bow which is the same on all different arrows. Since it accelerates less, it goes slower when it leaves the bow entirely m) The arrow gets kinetic energy, $1/2\ mv^2$, from the bow. The bow has potential energy stored in the stretched string. The bow also must get a little kinetic energy after the arrow leaves, because of conservation of linear momentum. There was zero velocity for the bow and for the arrow before the arrow went free. After the arrow is released, it has momentum in one direction. The bow must have equal and opposite momentum in the other direction to fulfill the conservation law. n) 30lb, but in the opposite direction, as Newton's third law states o) No, momentum must be conserved. Before the collision, all the momentum was eastward, and your car has all the momentum. After the collision, all the momentum must be east. Your car has ONLY eastward momentum and its momentum is less than before the accident. So it cannot cancel any northward momentum from the other car. Thus the other car can have only eastward momentum. [) 9.25m/s, by conservation of momentum q) Yes, but you can't tell how much. You can tell that there is SOME body damage by checking out the kinetic energy of the system before and after the collision. There is slightly less kinetic energy in the two cars after the collision than before. Since the total of kinetic and potential energy is conserved, there must have been a change in the amount of potential energy. It would go into squashing the cars. r) Make the ratio of the force between you and the Earth $F = \frac{Gm_{you}m_{earth}}{r_{earth}^2} = \text{your weight}$

and the force between you and the other planet is

$$F = \frac{Gm_{you}m_{planet}}{r_{planet}^2} = \frac{Gm_{you}\ \frac{1}{2}m_{earth}}{(\frac{1}{2}r_{earth})^2} = \text{your weight on new planet}$$

$$\frac{Gm_{you}\ \frac{1}{2}m_{earth}}{\frac{1}{4}r_{earth}^2} = \text{your weight on new planet}$$

$$2\frac{Gm_{you}m_{earth}}{r_{earth}^2} = \text{your weight on new planet}$$

$$2\ \text{weight on earth} = \text{your weight on new planet}$$

s) The less massive ball, the hollow one, goes faster t) As the ball unravels the spinning material goes from being closer to the center of the ball to being further. Conservation of angular momentum says the ball must spin slower because the radius is larger.